Advantages:

1. Make static components in optical systems more flexible, use the same optical components in different beam bath (e.g. satellites and space telescopes).
2. Distributed optical systems that has limited setup time and space (e.g. communication system).
3. Save time and energy spent for aligning manually.

Some optical assembling stations use rotational stages.

Most of self-aligning systems use wavefront sensors.

Align the system using camera without wavefront sensor:

1. More compact, cheaper
2. Easily redesign an optical system: changing fixed mounts to kinematic ones without splitting the beam.
3. No throughput loss or non-common path error.
State vector: \[ \mathbf{x} = [T_{xA}, T_{yA}, T_{xB}, T_{yB}]^T \]

Goal: Using only image data from CCD to correct tip-tilt misalignments of moving lenses A and B with respect to the beam.
Control Scheme

Plant & Kalman filter

Optical System (Lenses and Stages)

collimated laser beam

focused beam

images

stage command $u_k$

Kalman Filter

$F_k, B_k$

system function

$\hat{x}_{k|k-1}$

measurement function

$\hat{y}_k$

$y_k$

$K_c$

control gain

$\hat{x}_{k|k}$

$K_k$

Kalman gain

KL Modes Decomposition

$KL$ coefficients

Joyce Fang — State Estimation in Optical System Alignment Using Monochromatic Beam Imaging
Vector-mean-subtracted image vector

\[ \vec{r}_i = r_i - \mu(r_i) \quad \text{scan through } n \text{ images} \rightarrow \vec{R} = [\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n] \]
Principal Component Analysis
Karhunen-Loève

KL decomposition

Image-to-image covariance: \( S = \frac{1}{p-1} \bar{R}^T \bar{R} \)

Eigendecomposition: \( S \Phi = \Phi \Lambda \)

KL modes (eigen images): \( Z = \bar{R} \Phi \)
Karhunen-Loève Modal Reconstruction

KL coefficients and reconstruction

KL coefficients: \( w_i = Z_m^\dagger \bar{r}_i \)

Reconstructed image: \( \bar{c}_i = Z_m w_i \)

Simulated image

Reconstructed image

Subtracted image
Correlation Analysis

State feature extension: $x^{4\text{--elements to } 14\text{--elements}} \rightarrow t$

$$corr(T, W) = \frac{[T - \mu(T)][W - \mu(W)]^T}{\sigma(T)\sigma(W)^T}$$

<table>
<thead>
<tr>
<th>$T_xA$</th>
<th>$T_yA$</th>
<th>$T_xB$</th>
<th>$T_yB$</th>
<th>$T_xA^2$</th>
<th>$T_yA^2$</th>
<th>$T_xB^2$</th>
<th>$T_yB^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{xA}$</td>
<td>-7.2376e-06</td>
<td>-1.8364e-06</td>
<td>3.3744e-06</td>
<td>-5.0807e-06</td>
<td>-0.09891906</td>
<td>0.99182</td>
<td>-0.00010849</td>
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<tr>
<td>$T_{yA}$</td>
<td>-7.2838e-06</td>
<td>1.6838e-06</td>
<td>-3.6947e-06</td>
<td>-4.7556e-06</td>
<td>-0.99182</td>
<td>-0.00081838</td>
<td>0.00010112</td>
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<tr>
<td>$T_{yB}$</td>
<td>-2.6786e-07</td>
<td>1.4814e-07</td>
<td>9.2686e-07</td>
<td>-5.5158e-07</td>
<td>0.040765</td>
<td>3.3546e-05</td>
<td>-3.9133e-06</td>
</tr>
<tr>
<td>$T_{xA}^2$</td>
<td>-0.34279</td>
<td>2.6051e-07</td>
<td>-0.34408</td>
<td>-0.3439</td>
<td>1.3323e-05</td>
<td>-3.926e-05</td>
<td>3.0544e-07</td>
</tr>
<tr>
<td>$T_{yA}^2$</td>
<td>-0.34275</td>
<td>2.9209e-07</td>
<td>-0.34578</td>
<td>-0.3427</td>
<td>-3.8364e-05</td>
<td>1.5684e-05</td>
<td>1.8042e-07</td>
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<tr>
<td>$T_{xB}^2$</td>
<td>-0.61786</td>
<td>4.2899e-07</td>
<td>-0.60837</td>
<td>-0.6168</td>
<td>-5.2892e-07</td>
<td>4.4833e-05</td>
<td>-1.2233e-06</td>
</tr>
<tr>
<td>$T_{yB}^2$</td>
<td>-0.61778</td>
<td>4.4721e-07</td>
<td>-0.61146</td>
<td>-0.61382</td>
<td>4.4935e-05</td>
<td>-2.878e-06</td>
<td>-1.0476e-06</td>
</tr>
</tbody>
</table>

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Motivation  Model  Methods  Results  Conclusion

Model Fitting
Ridge linear regression

Normalized measurements
\[ y_i = \frac{1}{w_1} [w_2, w_4, w_5, w_6]^T \]

Ridge regression (10-fold cross-validation)
Design matrix: \( T_{\text{train}} \) \((14 \times n)\), Output matrix: \( Y_{\text{train}} \) \((4 \times n)\)
Model function: \( \hat{y}_i = Gt_i \xrightarrow{\text{nonlinear}} \hat{y}_i = h(x_i) \)

Normalized root mean square error (NRMSE)
\[ e_i = y_i - \hat{y}_i \]
\[ \text{NRMSE}(e_j) = \frac{\sqrt{\sum_{i=1}^{n} e_{ji}}/n}{y_{\text{max}} - y_{\text{min}}} \]

<table>
<thead>
<tr>
<th>\text{NRMSE}</th>
<th>( e_1 )</th>
<th>( e_2 )</th>
<th>( e_3 )</th>
<th>( e_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training Error</td>
<td>5.913e-3</td>
<td>5.916e-3</td>
<td>1.943e-2</td>
<td>1.961e-2</td>
</tr>
<tr>
<td>Test Error</td>
<td>5.966e-3</td>
<td>5.602e-3</td>
<td>1.448e-2</td>
<td>1.446e-2</td>
</tr>
</tbody>
</table>
State space representation

\[
\begin{align*}
\mathbf{x}_k &= F_k \mathbf{x}_{k-1} + B_k \mathbf{u}_k + \mathbf{q}_k \\
\mathbf{y}_k &= h_k(\mathbf{x}_k) + \mathbf{v}_k
\end{align*}
\]

Jacobian: \( H_k = \frac{\partial h}{\partial \mathbf{x}} |_{\mathbf{x}_{k|k-1}} \)
Kalman Filter Test in Simulation

State residuals

\[ q_k \sim N(0, Q_k) \text{ and } v_k \sim N(0, V_k) \]

- State 1 and 2 converge to \( \sim 9 \) arcsec (0.0025 degree).
- State 3 and 4 have similar precision but much lower accuracy \( \sim 65 \) arcsec (0.018 degree).
Kalman Filter Test in Simulation

Diagonal elements of covariance matrix $P$

- $P_{11}$ and $P_{22}$ decrease monotonically to a stable value.
- $P_{33}$ and $P_{44}$ decrease in the beginning, but start increasing after few steps.
$$\tilde{y} = y - \hat{y}$$

- Measurements converge to near zero $\rightarrow$ Kalman filter converges.
- Lens B is not fully observable using EKF with our 4 measurements $y_1$ to $y_4$. 
Measurement $y_2$ is symmetric with respect to both $x_3$ and $x_4$.

- Increase and decrease $x_3$ by $\Delta x_3$ → measurements $y_2^+$ and $y_2^-$.
- Calculate difference $\Delta y_2 = y_2^+ - y_2^-$ for correction.
Experimental Setup

Images collected in the experiment show large bias compared to our model → lens shift.

Test lens shift in ZEMAX: reconstructed image with mode $w_1$ to $w_4$

Projecting the remaining image onto mode coma-related modes will result in non trivial coefficients.
Conclusion

Future work - more complicated reconfigurable systems

1. Correct shift and tilt together in the experiment:
   - Aligning the lenses separately
     - Remove one lens and translate the other one along $z$-axis.
     - Might introduce stage error.
   - Estimating with a 8 DOF model
     - More measurements (e.g. spot center and coefficients).
     - Require more image data and computation time.

2. Improve the model describing optical system:
   - Higher order model functions
   - Learning methods other than linear regression
   - Different methods for modal decomposition (e.g. ICA)

3. Estimation and control:
   - Apply other estimation methods (e.g. Particle filter, Unscented Kalman filter)
   - A systematic control process will be built
Thank you