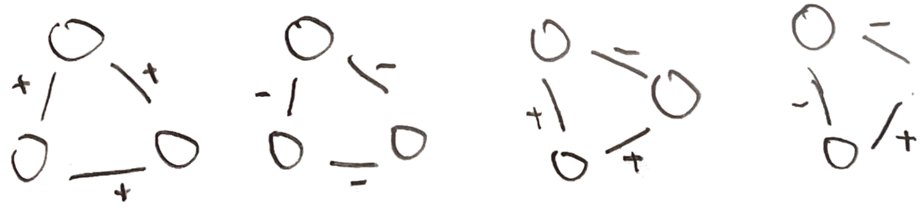
Goal: Come up with a function $G(n) :=$ the max number of groups which can be formed with n relationships

Trivial Case $n=2$. Possible relationships $+, -$. There are four possible triangles:



Balanced Unbalanced Unbalanced Balanced

This leads to max 2 alliances

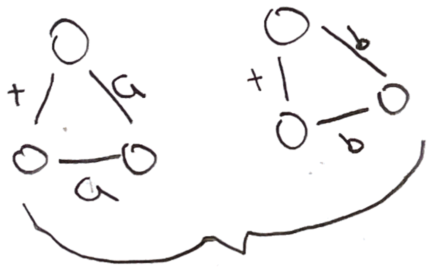
$n=3$. Have a positive relation $+$, and $(n-1)$ negative relations which are in a hierarchy: $+, a, b$. First, consider the single relationship triangles:

Balanced Unbalanced Unbalanced

Since we consider a,b to be "negative", these triangles are "equivalent" to

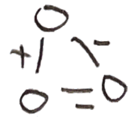
Now, consider the two-relationship triangles

First ones including positive:



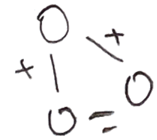
Balanced

equivalent to



Unbalanced

equivalent to



Next, ones with a,b

Unbalanced

Balanced

These decisions are arbitrary, here is the logic behind them: We put +, a, b in a hierarchy: + > a > b such that > means more positive. Since a is more positive than b, we treat a as + and b as - in triangles with only these relations.

Thus

Unbalanced Balanced

Now, consider 3 relationship triangles

Unbalanced

Arbitrarily, lets say unbalanced. There is no way to reduce this triangle to just + and - because a is - relative to + and a is + relative to b

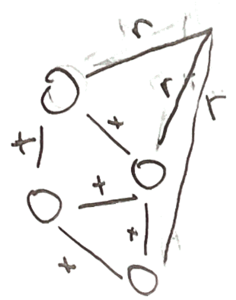
Note that for any n, the lowest relation in the hierarchy is negative relative to all other relations, thus it can be just considered a negative.

for $n=3$, $+\ a\ b \iff +\ a\ -$

for $n=4$, hierarchy could be $+\ a\ b\ -$

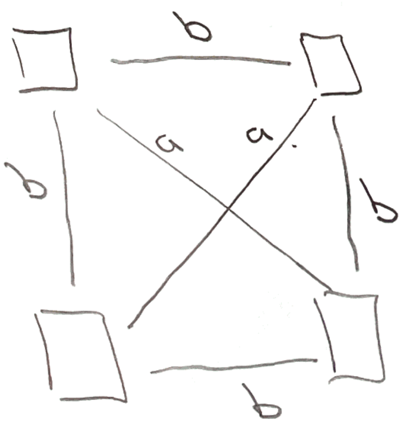
$\vdots$

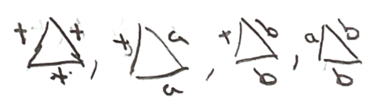
Considering $A(3)$

let □ = , □ = a group of nodes which all are + with each other and all have the same relationship with any external node

ie □ = allied group of nodes

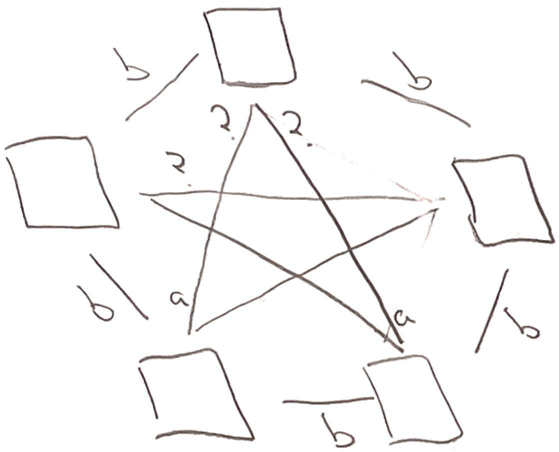
4 Groups is possible



Only triangles are 

all balanced

5 Groups is not balanced



No relationships for the final 3 connections are possible without creating an unbalanced triangle.

$\therefore G(3) = 4$

Theory: $G(1) = 1$ ← Because if only 1, only one alliance

$G(2) = 2$

$G(3) = 4$

$\vdots$

$G(n) = ?$

One equation which would work is $G(n) = 2^{n-1}$. If this is the correct equation, I would like to prove it, but not sure how.